

MA 3A	Mathematics Embedded Credit
Cape Career & Technology Center	Last Update: April 2017
Topic: Exponents, Square Roots, Scientific Notation	Focus: Basic Operations

Show-Me Standards: MA1, MA5	MO Grade Level Expectations: N1C8, N1C10, N2B9, N2C10, N1C7	NCTM Standards: 1A
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OBJECTIVE: Students will be able to complete basic operations involving exponents, square roots and scientific notation.

Introduction:

EXPONENTS:

Exponents are composed of two parts: the Base and the Exponent (or Power). The Base is acted on with the Exponent/Power by multiplication. The basic definition would state that the Base is multiplied by itself a number equal to the Exponent/Power.

EXAMPLES:


In the example below the Base is 2 and the Exponent/Power is 3. 2 is multiplied by itself three (3) times.

$$2^3 = 2 * 2 * 2 = 8$$


$$4^5 = 4 * 4 * 4 * 4 * 4 = 1024$$

In the example above the Base is 4 and the Exponent/Power is 5. 4 is multiplied by itself five (5) times.

EXPONENTS – Basic Operations:

 Any number raised to the 1st power is the number itself.

- $6^1 = 6$
- $a^1 = a$

 Any number, except zero, raised to the zero power is 1. Zero cannot be raised to the zero power.

- $125^0 = 1$
- $x^0 = 1$

 Any number written without an exponent is a number with an exponent of 1.

- $5 = 5^1$
- $b = b^1$

📖 When multiplying bases with exponents, **if the bases are the same**, add the exponents. Leave the base the same.

$$\begin{aligned} & x^a * x^b = x^{a+b} \\ \circ & 5^2 * 5^5 = 5^{2+5} = 5^7 \end{aligned}$$

📖 When dividing bases with exponents, **if the bases are the same**, subtract the bottom exponent from the top one. Leave the base the same.

$$\begin{aligned} & \frac{z^a}{z^b} = z^{a-b} \\ \circ & \frac{6^7}{6^3} = 6^{7-3} = 6^4 \\ & \frac{b^3}{b^5} = b^{3-5} = b^{-2} \end{aligned}$$

📖 Any base with a negative exponent may be rewritten as a fraction. The numerator in each case will be 1. The denominator will be the base with a **positive** exponent.

$$\circ a^{-3} = \frac{1}{a^3}$$

📖 When a negative exponent results in the answer for a given problem, give the final answer in fractional form.

$$\circ \frac{b^3}{b^5} = b^{3-5} = b^{-2} = \frac{1}{b^2}$$

📖 When multiplying different bases and the same exponents, multiply the bases and leave the exponent the same.

$$\begin{aligned} & a^y * b^y = (a * b)^y = (ab)^y \\ \circ & 2^3 * 3^3 = (2 * 3)^3 = 6^3 \end{aligned}$$

📖 When dividing different bases with the same exponents, divide the bases and leave the exponent the same.

$$\begin{aligned} & \frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a \\ \circ & \frac{4^3}{2^3} = \left(\frac{4}{2}\right)^3 = 2^3 = 8 \end{aligned}$$

📖 When exponents inside of a parenthesis are raised to a power, remove the parenthesis by **MULTIPLYING** the power of each base inside the parenthesis by the exponent outside the parenthesis.

$$(5^2)^3 = 5^{2*3} = 5^6$$

$$\circ (5a^3)^2 = (5^{1*2})(a^{3*2}) = 5^2 a^6$$

$$(a^2bc^3)^4 = (a^{2*4})(b^{1*4})(c^{3*4}) = a^8b^4c^{12}$$

📖 Fractional exponents can be written as **roots**. The numerator of the fraction is the power of the expression under the **radical sign**. The denominator is the amount given by the **index**.

$$\circ x^{\frac{1}{2}} = \sqrt{x}$$

$$(x+1)^{\frac{3}{2}} = \sqrt{(x+1)^3}$$

📖 To find the root of an algebraic expression raised to a power, simply change the expression to a fractional exponent and then reduce the fraction.

$$\sqrt{a^2} = a^{\frac{2}{2}} = a^1 = a$$

$$\sqrt{b^4} = b^{\frac{4}{2}} = b^2$$

$$\circ \sqrt[6]{z^4} = z^{\frac{4}{6}} = z^{\frac{2}{3}}$$

$$\sqrt{x+1} = (x+1)^{\frac{1}{2}}$$

$$\sqrt{4x^2} = (4x^2)^{\frac{1}{2}} = \left(4^{1*\frac{1}{2}}\right)\left(x^{2*\frac{1}{2}}\right) = 4^{\frac{1}{2}}x^{\frac{2}{2}} = 2x$$

SQUARE ROOTS:

The root is the number that is multiplied by itself to arrive at the power. The square root and the cube root are the most familiar, and commonly used, roots. The symbol for the root is called a **RADICAL SIGN** ($\sqrt{\quad}$). Under the radical sign is the number, or expression, whose root is to be found. When in this form, the expression is called **RADICALS**. In expressing the square root no number is necessary. When expressing the cube root, or greater, the **INDEX** is placed above the radical sign ($\sqrt[3]{\quad}$).

SQUARE ROOTS – Basic Operations:

📖 The small number written outside of the radical sign states the root to be found and is called the INDEX. The number must be a whole number, 2 or greater. The index is a positive number. No INDEX indicates that the root sought is the square root.

📖 Radicals may be written as powers, where the exponents are fractions. The index is converted to the denominator. The power of the number under the radical sign is converted to the numerator. The number under the radical sign becomes the base.

$$\sqrt{25} = (25)^{\frac{1}{2}}$$

○ $\sqrt[3]{64} = (64)^{\frac{1}{3}}$

$$\sqrt[3]{8^2} = (8)^{\frac{2}{3}}$$

📖 To find the square root of a number by inspection, determine what number can be multiplied by itself to equal the number inside the radical sign.

$$\sqrt{25} = 5, (5 * 5 = 25)$$

$$\sqrt{64} = 8, (8 * 8 = 64)$$

○ $\sqrt{121} = 11, (11 * 11 = 121)$

$$\sqrt{25^2} = 25, (25 * 25 = 25^2)$$

$$\sqrt{0.16} = 0.4, (0.4 * 0.4 = 0.16)$$

📖 The square root of a number can be either positive or negative. It is important to remember that two negative numbers multiplied together result in a positive answer. To indicate a positive or negative answer, use the following notation: \pm (plus or minus).

$$\sqrt{81} = \pm 9, (9 * 9 = 81, \text{ or } -9 * -9 = 81)$$

○ $\sqrt{35^2} = \pm 35, (35 * 35 = 35^2, \text{ or } -35 * -35 = 35^2)$

$$\sqrt{0.25} = \pm 0.5, (0.5 * 0.5 = 0.25, \text{ or } -0.5 * -0.5 = 0.25)$$

📖 To find the cube root of a number by inspection, determine what factor can be repeated three times to equal the number inside of the radical sign. Remember, it is possible to find the cube root of a negative number.

$$\sqrt[3]{216} = 6(6 * 6 * 6 = 216)$$

○ $\sqrt[3]{125} = -5(-5 * -5 * -5 = -125)$

$$\sqrt[3]{7^3} = 7(7 * 7 * 7 = 7^3)$$

SCIENTIFIC NOTATION:

A number expressed in scientific notation is a short-hand method of writing a very large, or very small, number. The decimal point is moved to follow the first significant figure. Then to maintain the place value of the original number, it is multiplied by some power of 10. Thus, the 'new' number retains its original value.

SCIENTIFIC NOTATION – Basic Operations:

WRITING *Scientific Notation* –

- 📖 Move the decimal point to follow the first significant figure. A number between 1 and 10 will be the result.
 - 📖 Count the number of places the decimal is moved; the number of moves is the power of 10.
 - If the decimal is moved to the **LEFT**, it is a **POSITIVE** power of 10.
 - If the decimal is moved to the **RIGHT**, it is a **NEGATIVE** power of 10.
 - 📖 Drop the zeroes that are no longer used as 'place holders'.
 - 📖 Write the number with the adjusted decimal place and show multiplication by 10 raised to the power.
- 📖 **EXAMPLE:** $2,003,000 = 2.003 \times 10^6$
 $0.009403000 = 9.403 \times 10^{-3}$

ADJUSTING IMPROPER *Scientific Notation* –

- 📖 Starting at the decimal point the count for the exponent begins with the given power.
 - 📖 Move the decimal point to follow the first significant figure.
 - 📖 Adjust the exponent to correspond with the movement of the decimal plus the original power.
 - 📖 Moving to the left is toward positive (larger) numbers, and to the right is toward negative (smaller) numbers.
 - 📖 REMEMBER: If the decimal number gets smaller by moving the decimal point, the exponent gets larger by the number of decimal places moved. If the decimal number gets larger by moving the decimal point, the exponent gets smaller by the number of decimal places moved.
- 📖 **EXAMPLES:** $432.005 \times 10^3 = 4.32005 \times 10^5$
 $0.0079004 \times 10^{-3} = 7.9004 \times 10^{-6}$

ADDITION and SUBTRACTION *in Scientific Notation* –

- 📖 To add/subtract numbers in scientific notation add/subtract the decimal numbers. In order to do so, the exponents in the powers of 10 must be the same. If they are not, adjust a decimal point and the corresponding exponent so that they are the same.
 - 📖 When the exponents are the same, align the decimals in a column and perform the calculations.
 - 📖 The powers of 10 remain the same in both addition and subtraction.
 - 📖 When finished with the arithmetic, adjust the answer to proper form.
- 📖 **EXAMPLES:** $4.5 \times 10^2 + 2 \times 10^2 = (4.5 + 2) \times 10^2 = 6.5 \times 10^2$
 $3.2 \times 10^2 - 1.7 \times 10^2 = (3.2 - 1.7) \times 10^2 = 1.5 \times 10^2$

MULTIPLICATION *in Scientific Notation* –

- 📖 To multiply numbers in scientific notation, multiply the decimal number together.
- 📖 Multiply the powers of 10 together by adding the exponents.
- 📖 Adjust the answer to proper scientific notation form.
- 📖 **EXAMPLE:** $4.5x10^3 * 2x10^4 = (4.5 * 2)x(10^3 * 10^4) = 9x10^{3+4} = 9x10^7$

DIVISION *in Scientific Notation* –

- 📖 To divide numbers in scientific notation, divide the decimal numbers.
- 📖 Divide the powers of 10 by subtracting the bottom (divisor) exponent from the top (dividend).
- 📖 Adjust the answer to proper scientific notation form.
- 📖 **EXAMPLE:** $\frac{3.12x10^6}{3x10^4} = \left(\frac{3.12}{3}\right)x\left(\frac{10^6}{10^4}\right) = 1.04x10^{6-4} = 1.04x10^2$

PROBLEMS:

ROOTS and EXPONENTS:

Find the Solution. On numbers 7 and 8, approximate the square root to 2 significant figures.

1. 6^2	2. 3^3	3. $(-8)^2$	4. $\sqrt{36}$
5. $\sqrt{121}$	6. $\sqrt[3]{-8}$	7. $\sqrt{20}$	8. $\sqrt{0.2}$

9. A square room is measured for wood flooring. The area of the room is 196 square feet. What length should the boards be if they are to be installed from one end of the room to the other without making a joint?

SCIENTIFIC NOTATION:

Complete the table below. Hint: convert numbers from scientific to standard notation before applying operation.

ADD:	$3.5 \times 10^4 + 62.1 \times 10^3$	$900 \times 10^{-6} + 450 \times 10^{-5}$
SUBTRACT:	$75 \times 10^3 - 0.32 \times 10^4$	$0.68 \times 10^{-2} - 24 \times 10^{-4}$
MULTIPLY:	$3 \times 10^5 * 0.4 \times 10^2$	$0.006 \times 10^{-2} * 50 \times 10^4$
DIVIDE:	$0.006 \times 10^{-2} \div 30 \times 10^4$	$45 \times 10^4 \div 15 \times 10^6$

- a. If a computer can perform an addition operation in 1.005×10^{-6} seconds, how long will it take the computer to perform 1000 addition operations.
- b. A building is constructed that is 1342 feet in height. Express this figure using units of miles. (One mile = 5,280 feet.)
- c. One foot of a steel pipe increases about 1.1×10^{-5} feet in length for a 1-degree-celsius rise in temperature. If a pipe of the same materials measures 10 feet long at 32°C , what does it measure at 42°C ? Give the answer in ordinary notation.